**Objective Function:**

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**Where:**

: volumetric energy rate ($/kWh)

: dynamic emissions-based economic signal ($/kWh), equal to marginal carbon emissions (metric tons CO2/MWh) \* Carbon Adder ($/metric ton) \* (1 MWh/1000 kWh)

: gross customer load (kW)

: time step length (hours)

: charging efficiency of the battery (%)

: degradation cost per battery charge-discharge cycle ($/cycle)

: nameplate energy capacity of the battery (kWh)

: power used to charge the battery (kW)

: discharging efficiency of the battery (%)

: power discharged from the battery (kW)

: energy level of the battery (kWh)

: non-coincident demand charge rate ($/kW)

: non-coincident maximum monthly demand (kW)

: coincident peak demand charge rate ($/kW)

: coincident peak maximum monthly demand (kW)

: coincident part-peak demand charge rate ($/kW)

: coincident part-peak maximum monthly demand (kW)

The objective function aims to minimize the total variable customer cost over the set time horizon. This cost has a number of components:

1. The net customer demand in each timestep (gross customer demand, minus power produced by solar if applicable, plus power consumed by the battery, minus power discharged from the battery) is multiplied by the volumetric energy rate.
2. The cost associated with a dynamic emissions-based economic signal, equal to the forecasted or real-time marginal carbon emissions rate multiplied by a carbon price.
3. The battery has a limited cycle life (assumed 10 years of daily cycling until the battery is degraded to 80% of its original capacity), so all power going into or out of the battery incurs a cost on the value of the asset. Note that this cycling penalty only applies to lithium-ion batteries; flow batteries do not experience cycling-related degradation in this way.
4. Depending on the rate structure, the customer may pay a non-coincident demand charge that is based on the highest 15-minute demand in each month. The customer may also pay up to two additional coincident demand charges: one based on the highest 15-minute demand occurring during peak hours, and another based on the highest 15-minute demand occurring during part-peak hours. For some tariffs, the peak or part-peak demand charge during winter months is $0/kW, some only have noncoincident demand charges, and some do not have any.
5. There is no cost associated with the energy level of the battery, but it must be included as a decision variable with an associated cost of 0 in order to be included in the constraint equations.

**Subject to the following constraints:**

**1.** The difference in the energy level of the battery between timesteps is equal to the power in minus the power out, with an efficiency penalty on both battery charge and discharge. This constraint applies for Timestep 1 through Timestep (N-1).

**2-5.** Power flowing into or out of the battery must be less than or equal to the rated power of the battery (or battery inverter), and greater than 0 kW. Power flowing into the battery includes both power from the PV system and power from the grid.

**6-7.** The energy level of the battery cannot be below 0 kWh, or above the nameplate energy capacity of the battery.

**8-9.** The initial state of charge/energy level of the battery is set to 30% (0.3 \* at the beginning of the year. The optimization algorithm will most likely discharge the battery as much as possible in the final timesteps of the time horizon to minimize the customer’s bill, a “greedy” dispatch action that would not be seen during continuous operation with an infinite time horizon. To create more realistic charge/discharge profiles, the time horizon (here, a calendar month or billing period) is “padded” with extra days at the end to ensure that the final energy levels are as close as possible to the values that would be seen during continuous operation. The initial state of charge in all months after the first is set equal to the final “unpadded” state of charge from the prior month to ensure continuity.

**10.** To include the demand charges while keeping the linear program optimization format (including a max() operator would make the problem nonlinear), an upper bound on net demand can be set as a decision variable, and then a constraint must be added to ensure that demand net of the storage system (and PV if applicable) is less than the optimally-set upper bound in all timesteps.

**11.** The above equation applies to the noncoincident demand charge, which applies to all timesteps in the month. This coincident peak demand charge applies only to timesteps that fall during peak and part-peak periods respectively.

**12.** The coincident part-peak demand charge applies only to timesteps that fall during part-peak timesteps.

**13.** If the solar plus storage systems is claiming the Investment Tax Credit for the storage system, it must be charged at least 75% from solar. However, the ITC amount is prorated by the amount of energy entering into the battery that comes from solar (ex. a storage system charged 90% from solar receives 90% of the ITC). As a result, the optimal amount of solar charging is likely higher than the minimum requirement of 75%, and likely very close to 100%. This formulation of the constraint requires that the storage system be charged 100% from solar.

**14.** If there is a no-export restriction, net load must be greater or equal to zero. This applies to storage-only systems only, because solar-plus-storage systems are allowed to export to the grid.

**15.** The Investor-Owned Utilities have suggested constraints on solar charging in particular hours as a proposed method for reducing greenhouse gas emissions associated with storage dispatch. Specifically, at least 50% of total charging would need to occur between 12:00 noon and 4:00 pm, and at least 50% of total discharging would need to occur between 4:00 pm and 9:00 pm.

Derivation of charging constraint in standard linear form Ax :

Derivation of discharging constraint in standard linear form Ax :

**16.** PG&E has suggested an alternative set of constraints on charging and discharging in particular hours as a proposed method for reducing greenhouse gas emissions associated with storage dispatch. Specifically, at least 50% of total charging would need to occur between 9:00 and 2:00 pm, and at least 50% of total discharging would need to occur between 4:00 pm and 9:00 pm (the Charging and Discharging Time Constraints). Charging would not be allowed to occur between 4:00 pm and 9:00 pm (the No-Charging Time Constraint).

Derivation of charging constraint in standard linear form Ax :

No-charging constraint in standard linear form Ax :

Note that because values of have been constrained to be non-negative, this is equivalent to the following constraint on for each timestep :

Derivation of discharging constraint in standard linear form Ax :

**17.** One alternative to setting a price on carbon would be to require that storage systems not increase emissions. Within the existing model structure, this can be achieved by adding the following Non-Positive GHG Emissions Impact constraint:

Note that here refers to the marginal emissions values (metric tons/MWh), and not the dynamic emissions signal ($/kWh). is equal to $0/kWh in all timesteps when this emissions-reduction approach is used, because the non-positive GHG emissions impact constraint is an alternative to setting a nonzero price associated with the marginal-emissions signal.

**18.** The GHG emissions solutions above can be supplemented with a requirement that the storage systems also obey an annual equivalent cycling constraint.

**19.** The GHG emissions solutions above can be supplemented with a requirement that the storage systems also obey an annual operational round-trip-efficiency constraint. Note that here includes any auxiliary or parasitic loads.

If the auxiliary/parasitic storage load is accounted for separately, this constraint can be written as:

**Non-Economic Solar Self-Supply Mode**

The economic-dispatch model above can be modified to encourage solar self-supply even when it is not economically beneficial, such as for a residential solar + storage customer on a non-time-of-use rate. The objective function provided to the linear-programming optimization algorithm is the same as above, but with a new term

That serves as a strong incentive for the storage system to minimize power produced by the solar PV system that is not stored in the battery. However, because PV production is not controllable (not a decision variable), this can be simplified to adding a cost term of

A value of 1 is currently used for , so this can be thought of as a $1.00/kWh incentive to charge the storage system as much as possible.

This new cost-function term alone is not sufficient to ensure that the solar-plus-storage system is self-consuming from solar, and not simply charging and discharging the battery to and from the grid. Two additional constraints must also be added.

**1.** First, an additional constraint is needed to ensure that the storage system only charges from excess solar. This can be achieved by creating a new vector with the excess solar production. This excess solar vector is equal to , with all negative values set to 0 kW. Then, power entering the storage system can be set less than or equal to excess solar production.

**2.** Secondly, a constraint needs to be added to ensure that the storage system only discharges when net load is positive, which means that the solar PV system is not fully meeting the customer load. This helps to ensure that the storage system does not discharge to the grid in addition to the solar system’s export, or charge and discharge simultaneously (which is physically impossible).

This can be represented similarly to the constraint above with the creation of a positive net load vector is equal to , with all negative values set to 0 kW. Then, power exiting the storage system can be set less than or equal to positive net load.